Final Exam

Math 321-A

Monday, December 10, 2001

For full credit show all work. You may use the `TI-89` to check your work, but I want to see all of the steps. You should be able to perform all integration by hand unless instructed otherwise. When in doubt, explain your reasoning.

1. Find the eccentricity of the following hyperbola.

$$\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$$

2. Find the slope of the curve $r = 2\cos(\theta)$ in polar coordinates at the point $\theta = \pi/3$.

3. Find the arc-length of the curve $\vec{r}(t) = \langle t^3 - 3t, 3t^2 \rangle$ for $-1 \leq t \leq 1$.

4. Find a unit vector in the same direction as the gradient for $f(x, y, z) = x + y^2 + z^3$.

5. Find the acute angle between the following two planes:

$$\begin{aligned} 3x - 4y + 5z &= 2 \\ x - 2y - 3z &= 0 \end{aligned}$$

6. Either find an intersection point for the following two lines or show that none exists.

$$\begin{aligned} \vec{r_1}(s) &= < 3s + 1, -2s + 5, s > \\ \vec{r_2}(t) &= < 2t - 1, -t, -t > \end{aligned}$$

7. Find the perpendicular distance from the point $(1, 3, 5)$ to the following plane:

$$x + y + z = 4$$

8. Find the unit tangent, normal, and binormal vectors ($\vec{T}$, $\vec{N}$, and $\vec{B}$) for the following curve:
$$\vec{r}(t) =< t, \sin t, \cos t >$$

9. Use partial derivatives to find y' implicitly where $x^2y + xy^2 = 12$.

10. Find any critical points for the following function and use the second partials test on them:
$$f(x, y) = ye^x + 3e^x - y$$

11. Find the surface area of the surface $\vec{r}(u, v) =< u + v, u - v, v - u >$ where $0 \leq u \leq 1$ and $2 \leq v \leq 3$.

12. Find a potential function for the vector field $\vec{F}(x, y) = \langle 2xe^y, x^2e^y + 3y^2 \rangle$.

13. Find curl $\vec{F}$ where $\vec{F}(x, y, z) = \langle x + y + z, x^2, y^2 \rangle$.

14. Why did we use polar coordinates instead of Cartesian coordinates when showing that the orbits of planets using an inverse square law for gravity had to be conic sections?

15. Find the curvature of the curve $\vec{r}(t) = \langle t^2, t^3, t^4 \rangle$ when $t = 1$.

16. Find the center of gravity for the planar region bounded by $y = \sqrt{x}$ and $x = 4$ in the first quadrant.

17. Integrate $f(x, y) = 2x + 3y$ along the curve $\vec{r}(t) =< \cos t, \sin t >$ for $0 \leq t \leq \pi$.

18. Integrate in spherical coordinates $f(\rho) = \rho$ over the upper half of the unit ball (where $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$).

19. Find the normal and tangential components of acceleration for $\vec{r}(t) = \langle e^t, e^{-t} \rangle$.

20. Given $f(x, y) = x^2 - y^2$ and $\vec{u} = \langle \sqrt{3}/2, -1/2 \rangle$, find the directional derivative $D_{\vec{u}} f(x, y)$.