Exam #4 Solutions

Math 115-A

Wednesday, December 1, 2004

1.

$$\begin{aligned}\frac{12\left(\cos\left(85^\circ\right)+i\sin\left(85^\circ\right)\right)}{4\left(\cos\left(20^\circ\right)+i\sin\left(20^\circ\right)\right)} &= \frac{12}{4}\left(\cos\left(85^\circ-20^\circ\right)+i\sin\left(85^\circ-20^\circ\right)\right)\\ &= 3\left(\cos\left(65^\circ\right)+i\sin\left(65^\circ\right)\right)\end{aligned}$$

2. We'll apply DeMoivre's Theorem after converting $1-i$ to trigonometric form. Note that $1-i$ is in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{1^2+(-1)^2}\\ &= \sqrt{2}\\ \tan(\theta) &= \frac{-1}{1}\\ &= -1\\ \theta &= \tan^{-1}(-1)\\ &= -45^\circ\\ (1-i)^{10} &= \left(\sqrt{2}\right)^{10}\left(\cos\left(10(-45)^\circ\right)+i\sin\left(10(-45)^\circ\right)\right)\\ &= 2^5\left(\cos\left(-450^\circ\right)+i\sin\left(-450^\circ\right)\right)\\ &= 32(0+i(-1))\\ &= -32i\end{aligned}$$

3.

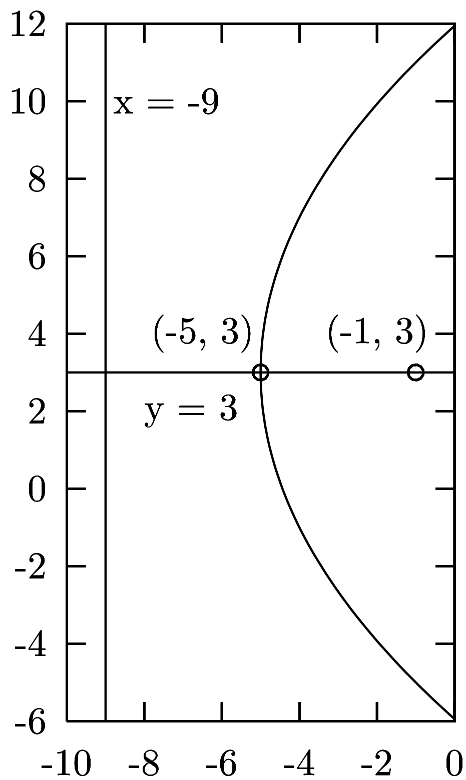
$$\begin{aligned}(3-5i)(2+7i) &= 6+21i-10i-35i^2\\ &= 6+11i-35(-1)\\ &= 41+11i\end{aligned}$$

4. The vertex is at $(-5,3)$. $16=4p$ implies that $p=4$. Since the y term is squared, the parabola opens leftward or rightward. Since $p>0$, the parabola opens to the right.

   The focus is at $(-5+4,3)=(-1,3)$.

   The directrix is a vertical line $p=4$ units to the left of the vertex, i.e., at $x=-5-4=-9$.

   The axis is the horizontal line through the vertex and the focus, i.e., at $y=3$.



5. We first convert $8i$ to trigonometric form. Then we take the square root of r and halve θ to find one square root. The second square root is $360°/2 = 180°$ away. We finish by converting the roots to standard form.

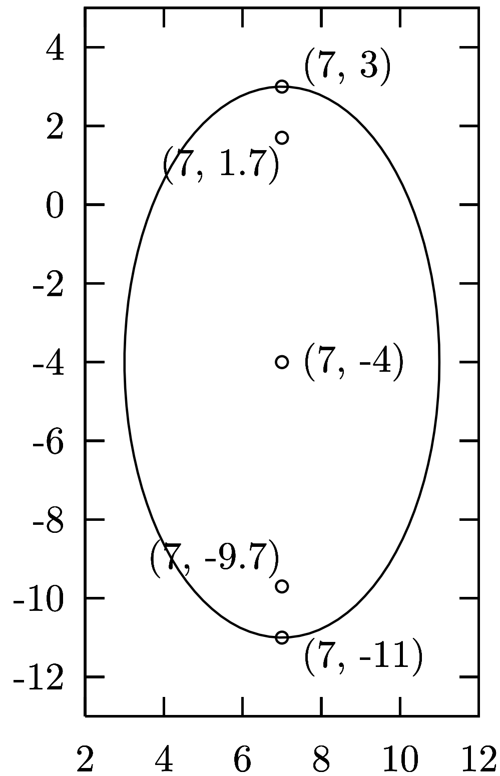
$$\begin{aligned}
r &= 8 \\
\theta &= 90° \\
\text{First root: } \sqrt{8}\left(\cos\left(\frac{90°}{2}\right) + i\sin\left(\frac{90°}{2}\right)\right) &= \sqrt{8}\left(\cos(45°) + i\sin(45°)\right) \\
&= \sqrt{8}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) \\
&= 2 + 2i \\
\text{Second root: } \sqrt{8}\left(\cos(45° + 180°) + i\sin(45° + 180°)\right) &= \sqrt{8}\left(\cos(225°) + i\sin(225°)\right) \\
&= \sqrt{8}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \\
&= -2 - 2i
\end{aligned}$$

6. The center is at $(7, -4)$.

   $a^2 = 49$ and $b^2 = 16$, so $a = 7$ and $b = 4$. The ellipse has a vertical major axis.

   The vertices are 7 units below and above the center at $(7, -11)$ and $(7, 3)$ respectively.

   $c = \sqrt{a^2 - b^2} = \sqrt{49 - 16} = \sqrt{33}$. The foci are c units below and above the center, at $(7, -4 - \sqrt{33}) = (7, -9.7)$ and $(7, -4 + \sqrt{33}) = (7, 1.7)$ respectively. The eccentricity is $e = c/a = \sqrt{33}/7 = 0.82$.

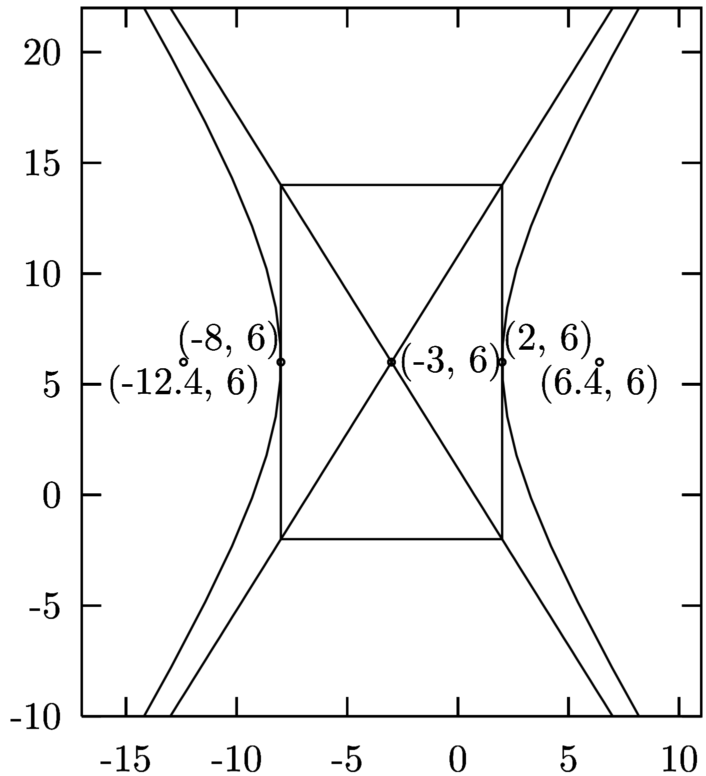


7. The center is at $(-3,6)$.

   $a^2 = 25$ and $b^2 = 64$, so $a = 5$ and $b = 8$. The hyperbola opens to the left and to the right.

   The vertices are 5 units to the left and to the right of the center at $(-8,6)$ and $(2,6)$ respectively.

   $c = \sqrt{a^2+b^2} = \sqrt{25+64} = \sqrt{89}$. The foci are c units to the left and to the right of the center, at $(-3-\sqrt{89},6) = (-12.4,6)$ and $(-3+\sqrt{89},6) = (6.4,6)$ respectively. The eccentricity is $e = c/a = \sqrt{89}/5 = 1.89$.



8. Parabolas serve as transitions from ellipses to hyperbolas. Since the eccentricity of any ellipse is less than 1 and the eccentricity of any hyperbola is greater than 1, it makes sense to set the eccentricity of a parabola to 1.

9. We multiply the numerator and denominator of the fraction by the conjugate of the denominator, simplify, and write in standard form.

$$\begin{aligned}
\frac{3+5i}{7-2i} &= \left(\frac{3+5i}{7-2i}\right)\left(\frac{7+2i}{7+2i}\right) \\
&= \frac{21+6i+35i+10i^2}{49-4i^2} \\
&= \frac{21+41i+10(-1)}{49-4(-1)} \\
&= \frac{11+41i}{53} \\
&= \frac{11}{53}+\frac{41}{53}i
\end{aligned}$$

10.

$$
\begin{aligned}
3x^2 + 5x + 7 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-5 \pm \sqrt{5^2 - 4(3)(7)}}{2(3)} \\
&= \frac{-5 \pm \sqrt{25 - 84}}{6} \\
&= \frac{-5 \pm \sqrt{-59}}{6} \\
&= \frac{-5 \pm \sqrt{59}i}{6} \\
&= -\frac{5}{6} \pm \frac{\sqrt{59}}{6} i
\end{aligned}
$$