

Mathematica Practice

Math 321-A

Friday, October 20, 2006

Working with Gradients

To work with gradients and related operations, you need to import the following package:

```
<<Calculus`VectorAnalysis`
```

Before you can do any work, you need to tell *Mathematica* what coordinates system (rectangular, polar, cylindrical, spherical, etc.) you are using.

```
SetCoordinates[Cartesian[x, y, z]]
```

Then you can compute a gradient with the function `Grad`.

For example:

```
Grad[Cos[x*y] + Tan[y - z]]
```

PlotField

To plot a two-dimensional field of gradients, first import the following package:

```
<<Graphics`PlotField`
```

Then we can use the command `PlotGradientField` to plot a gradient field.

```
graph1 = PlotGradientField[x^2 + y^2, {x, -3, 3}, {y, -3, 3}]
```

Remember the `VectorHeads->True` option if you want to emphasize the direction of the vectors.

You can see the effect of the gradient by superimposing the graph of the gradients with the result of a contour plot.

```
graph2 = ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3}]  
Show[graph2, graph1]
```

Note that because of the shading of the contour plot, it is important to place it before the gradient field in the `Show` command.

PlotField3D

We can perform the analogous work with a function of three variables, using '3D' where necessary.

```
<<Graphics`PlotField3D`  
graph1 = PlotGradientField3D[x^2 + y^2 + z^2, {x, -3, 3}, {y, -3, 3}, {z, -3, 3}]
```

Practice

1. Compute the gradient of $\tan^{-1}(x/y + y/z + z/x)$.
2. Superimpose the gradients on the contour plot of $x^2 - y^2$ for $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$.
3. Superimpose the gradients on the contour plot of $(x^2 - y^2)/(x^2 + y^2)$ for $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$. (You'll get some warnings about division by 0 at the origin, but you can ignore them.)
4. Create the gradient field for $f(x, y, z) = xyz$ for all three variables lying between -1 and 1 .