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by the Elon University Honor Code

Name ______________________ Signature ______________________

Exam #1

Math 425-A

Friday, September 28, 2007

For full credit explain everything.

1. Give an example of a sequence of rational numbers satisfying the Cauchy criterion that does not converge to a rational limit. (Be sure to prove both of those two properties.)
2. Give an example of a theorem from class that depends on the Axiom of Completeness, i.e., that is not true for the rational numbers.
3. Prove that the sequence defined by $a_1 = 1$ and $a_{n+1} = (a_n + 10)/2$ converges, and find its limit.
4. Show that the sequence (a_n) where $a_n = \cos(n)$ has a convergent subsequence.
5. Explain the difference between a series and its limit (if it converges).
6. Let S be a non-empty set of real numbers that is bounded above, and let L be its supremum. Use Lemma 1.3.7 and the Squeeze Theorem to show that there is a sequence of elements in S that converges to L. (Let $\epsilon = 1/n$ for each n.)
7. Prove that if a non-empty set S of real numbers is bounded above then it has an infinite number of upper bounds.
8. Use the Algebraic Limit Theorem to find the limit of the following sequence, articulating each step. You may assume that $1/n \to 0$ and $1/n^2 \to 0$.
$$\left(\frac{2n^2 + 5n + 2}{3n^2 - 4n + 1}\right)$$
(You should divide the numerator and denominator by the appropriate power of n before using the Algebraic Limit Theorem.)
9. Given that between any two distinct real numbers there exists a rational number, show that between any two distinct real numbers there exists an irrational number.
10. Give an example of two convergent sequences (a_n) and (b_n) such that $a_n < b_n$ but $\lim a_n = \lim b_n$.