Solutions to Final Exam

Math 111-C

Thursday, December 9, 1999

1. The parabola opens upward, so the curve has not been reflected across the x-axis. The vertex is $(-3, 8)$, so the graph has been shifted left three units and up eight units from the graph of $y = x^2$:

$$\begin{array}{rrcl} \text{Original graph:} & y & = & x^2 \\ \text{Shifted left by 3:} & y & = & (x+3)^2 \\ \text{Shifted up by 8:} & y & = & (x+3)^2 + 8 \\ & & = & x^2 + 6x + 17 \end{array}$$

The equation of the graph is $y = x^2 + 6x + 17$.

2. We clear the fractions before solving the equation:

$$\begin{array}{rrcl} & \dfrac{2}{x} + \dfrac{3}{x+4} & = & \dfrac{13}{5} \\ \text{Multiply by } x\text{:} & 2 + \dfrac{3x}{x+4} & = & \dfrac{13x}{5} \\ \text{Multiply by } x+4\text{:} & 2(x+4) + 3x & = & \dfrac{13x(x+4)}{5} \\ \text{Multiply by 5:} & 10(x+4) + 15x & = & 13x(x+4) \\ & 10x + 40 + 15x & = & 13x^2 + 52x \\ & 13x^2 + 27x - 40 & = & 0 \\ & x & = & \dfrac{-27 \pm \sqrt{27^2 - 4(13)(-40)}}{2(13)} \\ & & = & \dfrac{-27 \pm \sqrt{729 + 2080}}{26} \\ & & = & \dfrac{-27 \pm \sqrt{2809}}{26} \\ & & = & \dfrac{-27 \pm 53}{26} \end{array}$$

$$
\begin{aligned}
&= -\frac{80}{26} \quad \text{or} \quad \frac{26}{26} \\
&= -\frac{40}{13} \quad \text{or} \quad 1 \\
&= -3.077 \quad \text{or} \quad 1
\end{aligned}
$$

3. We use the equation for a circle:

$$
\begin{aligned}
(x-h)^2 + (y-k)^2 &= r^2 \\
(x-(-1))^2 + (y-4)^2 &= 5^2 \\
(x+1)^2 + (y-4)^2 &= 25
\end{aligned}
$$

4. The inequality is of the form $a^x > b^x$ where $a < b$. This can only be true for negative x, and the solution set is the interval $(-\infty, 0)$.

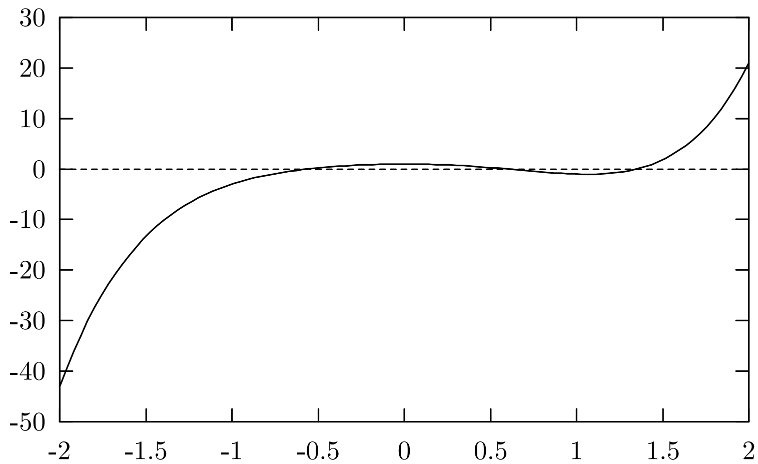
5. Parallel lines have the same slope, so the line we seek has slope 2 and y-intercept $(0, -5)$. The point-slope form of the equation of the line is $y = 2x - 5$.

6. We move all the terms to one side:

$$
\begin{aligned}
x(x+1) &\leq 2 \\
x^2 + x &\leq 2 \\
x^2 + x - 2 &\leq 0 \\
(x+2)(x-1) &\leq 0
\end{aligned}
$$

   The parabola $y = x^2 + x - 2$ has roots at -2 and 1, and opens upwards. $(x+2)(x-1) \leq 0$ is true between the roots, inclusive, and the solution is $[-2, 1]$.

7. The graph of $y = x^5 - 3x^2 + 1$ is as follows:



The three roots are at -0.561, 0.599, and 1.348.

8. We use the quadratic formula:

$$\begin{aligned}
3x^2 + 5x - 7 &= 0 \\
x &= \frac{-5 \pm \sqrt{5^2 - 4(3)(-7)}}{2(3)} \\
&= \frac{-5 \pm \sqrt{25 + 84}}{6} \\
&= \frac{-5 \pm \sqrt{109}}{6} \\
&= -2.573 \quad \text{or} \quad 0.907
\end{aligned}$$

9. Because it is an inequality, we can't cross-multiply. Instead, we combine all of the terms on one side:

$$\begin{aligned}
\frac{2}{x} &< \frac{3}{x+1} \\
0 &< \frac{3}{x+1} - \frac{2}{x} \\
0 &< \frac{3x}{x(x+1)} - \frac{2(x+1)}{x(x+1)} \\
0 &< \frac{x-2}{x(x+1)}
\end{aligned}$$

	$(-\infty,-1)$	$\{-1\}$	$(-1,0)$	$\{0\}$	$(0,2)$	$\{2\}$	$(2,\infty)$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\frac{x-2}{x(x+1)}$	$-$	undef	$+$	undef	$-$	0	$+$

The solution is $(-1,0)\cup(2,\infty)$.

10. If $f(x)$ is the function, simplify $f(-x)$. If the result is $-f(x)$, the function is odd. If the result is $f(x)$, the function is even. Otherwise the function is neither.

    If $f(x)$ is a polynomial, it is faster to look at the exponents of the terms. If they are all odd, the function is odd. If they are all even, the function is even. Otherwise the function is neither.

11. The midpoint is:

$$\begin{aligned}\left(\frac{3+(-2)}{2},\frac{4+7}{2}\right) &= \left(\frac{1}{2},\frac{11}{2}\right)\\ &= (0.5,5.5)\end{aligned}$$

12. The distance is:

$$\begin{aligned}\sqrt{(-2-3)^2+(7-4)^2} &= \sqrt{(-5)^2+3^2}\\ &= \sqrt{25+9}\\ &= \sqrt{34}\\ &= 5.831\end{aligned}$$

13. We first solve for the coordinates of the vertex:

$$\begin{aligned}h &= -\frac{b}{2a}\\ &= -\frac{1}{2(3)}\\ &= -\frac{1}{6}\\ k &= \frac{4ac-b^2}{4a}\\ &= \frac{4(3)(-7)-1^2}{4(3)}\end{aligned}$$

$$
\begin{aligned}
&= \frac{-84-1}{12} \\
&= -\frac{85}{12} \\
y &= 3\left(x - \left(-\frac{1}{6}\right)\right)^2 + \left(-\frac{85}{12}\right) \\
&= 3\left(x + \frac{1}{6}\right)^2 - \frac{85}{12}
\end{aligned}
$$

14. There is no cancellation in $y = (3x+2)/(7x-1)$.

    The x-intercept occurs when the numerator is zero:

$$
\begin{aligned}
3x + 2 &= 0 \\
3x &= -2 \\
x &= -\frac{2}{3} \\
&= -0.667
\end{aligned}
$$

    The y-intercept occurs when $x = 0$, i.e., at $y = 2/(-1) = -2$.

15. We split the absolute value inequality up into two linear inequalities:

$$
\begin{array}{rcl}
|2x-5| & \geq & 3 \\
2x - 5 \leq -3 & \text{or} & 3 \leq 2x - 5 \\
2x \leq 2 & \text{or} & 8 \leq 2x \\
x \leq 1 & \text{or} & 4 \leq x
\end{array}
$$

    and the solution is $(-\infty, 1] \cup [4, \infty)$.

16. Using the point-slope form for the equation of a line:

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 7 &= -\frac{1}{2}(x-5) \\
y &= -\frac{x}{2} + \frac{19}{2} \\
&= -0.5x + 9.5
\end{aligned}
$$

17. (a) A rational function has a slant asymptote only if the degree of the numerator is one larger than the degree of the denominator. When that happens, the quotient is a linear function and is the slant asymptote.

(b)

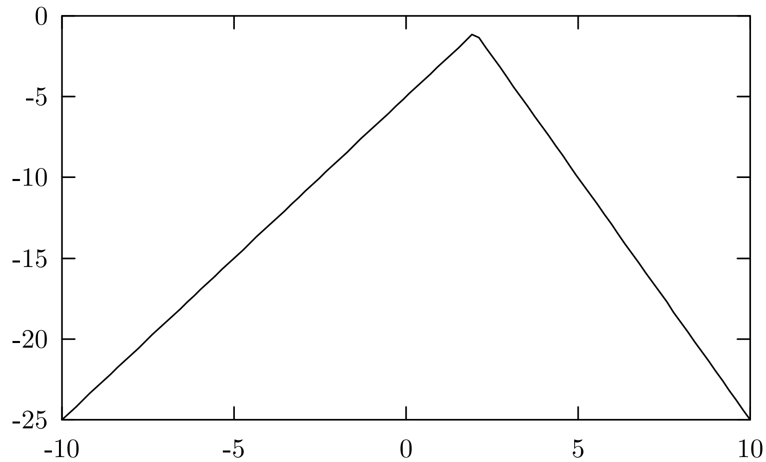
$$
\begin{array}{rrrr}
 & & 3x & +13 \\ \cline{2-4}
x \quad -5 & \multicolumn{1}{|r}{3x^2} & -2x & +4 \\
 & 3x^2 & -15x & \\ \cline{2-3}
 & & 13x & +4 \\
 & & 13x & -65 \\ \cline{3-4}
 & & & 69
\end{array}
$$

and the slant asymptote is $y = 3x + 13$.

18. We use the formulae for h and k:

$$
\begin{aligned}
h &= -\frac{b}{2a} \\
&= -\frac{1}{2(-4)} \\
&= \frac{1}{8} \\
k &= \frac{4ac - b^2}{4a} \\
&= \frac{4(-4)(-7) - 1^2}{4(-4)} \\
&= \frac{112 - 1}{-16} \\
&= -\frac{111}{16} \\
\text{vertex} &= (h, k) \\
&= \left(\frac{1}{8}, -\frac{111}{16}\right) \\
&= (0.125, -6.938) \\
\text{axis:} \quad x &= h \\
&= \frac{1}{8} \\
&= 0.125
\end{aligned}
$$

19. The graph is as follows:



20. We write in what $f(x)$ does, and then what $g(x)$ does.

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= (g(x))^2 + 3\,(g(x)) \\
&= (2x-5)^2 + 3(2x-5) \\
&= 4x^2 - 10x - 10x + 25 + 6x - 15 \\
&= 4x^2 - 14x + 10
\end{aligned}
$$