Take-Home Exam #3

Math 425-A

Due in Class: Tuesday, December 7, 1999

You may consult any inanimate reference, e.g., your text, your notes, and library texts. You are honor-bound not to consult your class-mates, other faculty members, or any other person.

1. Find the derivative of $f(x) = \sqrt[3]{x}$ using the definition of the derivative. Also determine where the derivative is defined. Note: $a - b = \left(\sqrt[3]{a} - \sqrt[3]{b}\right)\left((\sqrt[3]{a})^2 + \sqrt[3]{ab} + \left(\sqrt[3]{b}\right)^2\right)$. You may assume that $f(x)$ is continuous.

2. Use the Mean Value Theorem to show that a differentiable periodic function must have an infinite number of critical points. (A function $f : \mathbb{R} \to \mathbb{R}$ is periodic if there is a number $a > 0$ such that $f(x + a) = f(x)$ for all x.)

3. Use l'Hôpital's Rule to find the following limit. Be sure to verify explicitly that the limit satisfies the conditions of l'Hôpital's Rule.

$$\lim_{x \to 3} \frac{\cos \pi x + 1}{\sin \pi x}$$

4. Let $f(x) = x^3 + 9x - 5$.
   (a) Use $f'(x)$ to show that $f(x)$ is invertible.
   (b) Find the tangent line to the curve $y = f^{-1}(x)$ at the point $(21, 2)$.

5. Let $f(x) = x^2 + x$ on the interval $[a, b] = [1, 10]$. Find a partition P such that $U(P, f) - L(P, f) < \epsilon$ with $\epsilon = 0.001$.