

# Giving Students Their Own Curves in Multivariable Calculus

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March 11–12, 2005

There is a great deal of technique that we cover in my multivariable calculus classes. I have trouble keeping up with all of it, as do my students. In order for all of us to succeed, I assign and grade a good deal of work.

Many of the problem set questions that I assign never reach deeper than the given technique that they assess, so I also assign monthly projects that I hope will engage the students on a deeper level.

My projects have the following goals, aside from assessing mastery of relevant skills:

- They must be visual.
- They must be tangible.
- They should be varied in some way from student to student, so that the class would have some interest in how each other's work turned out.
- In spite of the variation, they should all be graded with a consistent rubric.

The focus of the monthly projects is on curves, both in the plane and in three-dimensional space. Each student is assigned their own curve at the beginning of the semester to work with.

The curves need to have some features of interest; I have relied heavily upon *A Catalog of Special Plane Curves* by J. Dennis Lawrence in finding suitable choices. Typically I have less than twenty students in my section, so I have accumulated a repository of twenty or so different parametric curves that I use from semester to semester.

My students are nearly as bad as I am in drawing pictures. Fortunately none of us need to do so by hand; we use *Mathematica* pretty thoroughly in our class, and I ask the students to rely upon *Mathematica* for their graphs.

*Mathematica* (and computer algebra systems in general) free us from having to restrict our range of examples to those that can be done by hand.

## Project #1

Curvature is a way of measuring how fast a curve bends at a given point. One common intuitive use is in driving a car: you should decelerate going into a curve, and accelerate coming out of a curve.

Going into and coming out of a curve is a measure of curvature: curvature is increasing as you go into a curve, and it is decreasing as you come out of the curve. The moment when you take your foot off the brake and start to accelerate is at the maximum of the curve's curvature. Conversely, at the minimum of a curve's curvature is where you should ease up on the accelerator and start to brake. (This assumes that there is no other traffic, etc.)

## Comments on Project #1

- The students can see on there curves where the maximal and minimal curvatures occur.
- The problem is tangible: all of my students have driven a car.
- The different curves yield different answers which are easy for them to demonstrate to each other.
- It is very quick to see if the students answers match their curves.

## Project #2

In this project you will take the curve that you worked with in your first project and use it to create a surface.

Add the parameter  $t$  as a third coordinate to your curve to make it a three-dimensional curve. Plot the new curve and find its arc-length.

Connect the new curve to the  $z$ -axis by constructing the surface  $(s \cdot x(t), s \cdot y(t), t)$  where  $0 \leq s \leq 1$ . Plot this surface, and find its surface area.

Construct a graph of ten normal vectors to this surface along the curve  $s = 1/2$ .

## Comments on Project #2

- The students can see how their curves can be stretched into “spiral staircases”.
- The pictures are great to play with; I animate them on my web page so they can rotate them in various ways.
- The different curves yield different answers which are easy for them to demonstrate to each other.
- It is very quick to see if the students have followed the instructions.

## Project #3

*Pappus Theorem* states that for a surface of revolution, the surface area is the product of the length of the curve with the distance traveled by the centroid, i.e., the length of the curve with  $2\pi$  times the distance from the centroid to the axis of revolution.

In this project you will verify Pappus Theorem for your curve as it is rotated around the  $x$ -axis. (To ensure that your curve does not intersect the  $x$ -axis, I may have moved it upward by adding a positive constant to the  $y$ -coordinate as well as to the  $x$ -coordinate.)

## Comments on Project #3

- The students can see the surfaces of revolution.
- The problem is tangible: they can see the surface whose area they are computing.
- The different curves yield different answers which are easy for them to demonstrate to each other.
- It is very quick to see if the students answers match their curves: in particular the two surface area computations should match.

I assign monthly projects in most of my classes, trying to get the students to apply the course's contents to deeper problems than I can ask about on a weekly basis.

Whenever possible I like to be able to have the students have some ownership of their work, some sense that what they produce is not the same as what every other student produces in the class.

If I'm not careful, I can end up with increasing the amount of grading I do by one or two orders of magnitude. In this presentation I've showed three assignments in multivariable calculus, that with the help of *Mathematica* are useful to the students and viable for me to grade.

This presentation is available from my home page

<http://frodo.elon.edu>

under the link *Presentations*.

Reference:

Lawrence, J. Dennis, *A Catalog of Special Plane Curves*. New York: Dover.