Using Art to Teach Elementary Group Theory

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M. C. Escher (Maurits Cornelis Escher, 1898–1972) enjoyed a long career, creating wood-prints that played with our perceptions, celebrating symmetry and paradox. I first encountered Escher's work in the pages of Martin Gardner's *Mathematical Games* column in *Scientific American* and quickly fell in love with it.

Now that I am a mathematics teacher, I fall back on Escher's work when I need non-trivial examples of symmetries. In this presentation I will discuss a particular classroom use of his wood-prints, namely conveying some notions of elementary abstract algebra to freshmen.

This talk is available from the web site

`http://frodo.elon.edu`

under the link *Presentations*.

In the Fall of 2003 and again in the Fall of 2004, I was fortunate enough to teach an honors section of our Calculus with Analytic Geometry I class. This class contained between fifteen and twenty of our brighter students, not all of whom were considering a major in mathematics.

Our department has for a number of years been considering ways of revising our major curriculum, particularly with the intent of exposing freshmen and sophomores to topics that they will see in their junior and senior years, in the hopes of laying down a foundation. As an algebraist, I have been interested in showing students what I can of group actions, building on their prior knowledge of symmetries of graphs.

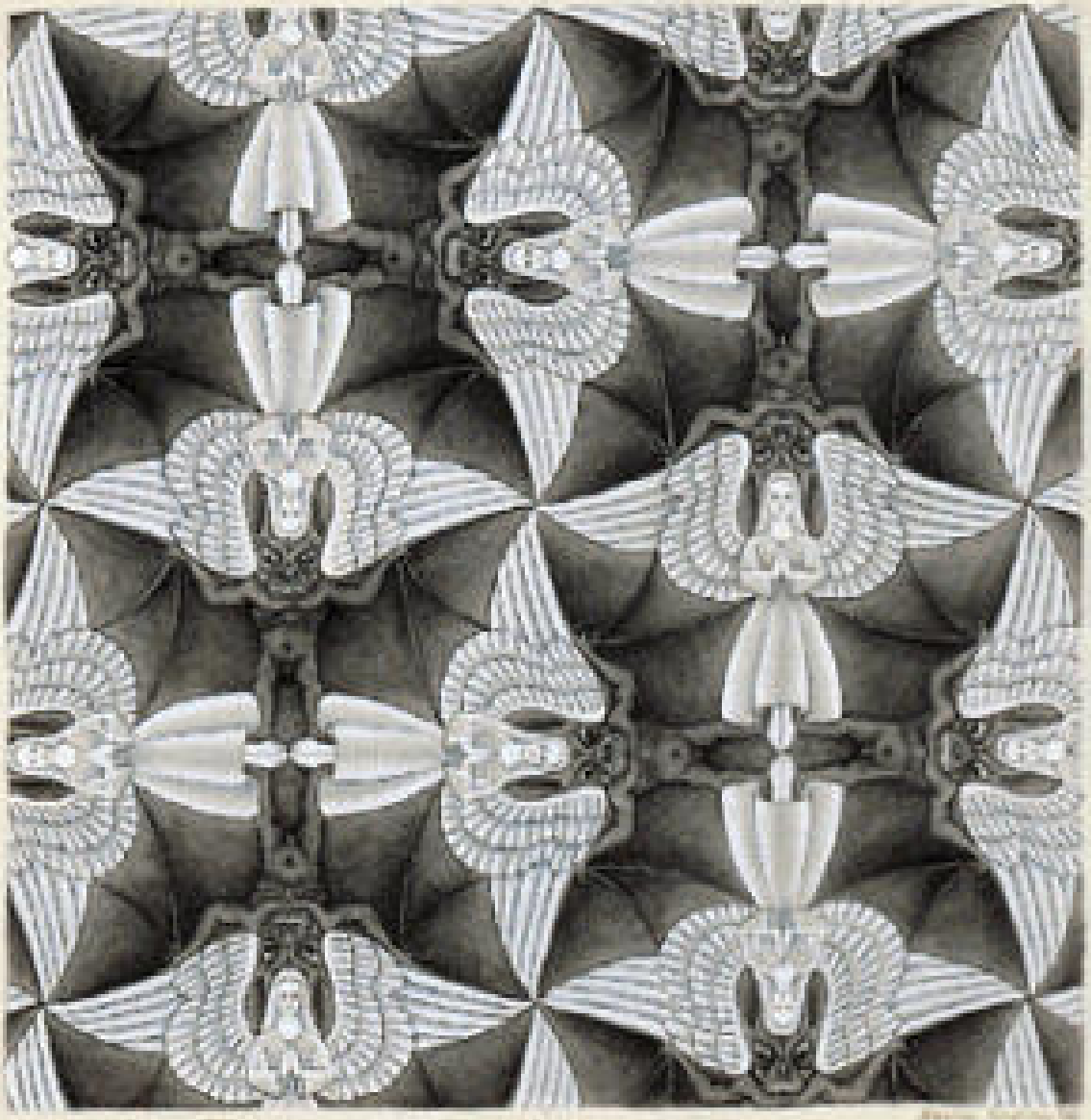
Given the opportunity to teach these honors sections, I have incorporated more history, discrete modeling, and discussions of symmetries than I normally do in a first calculus class.

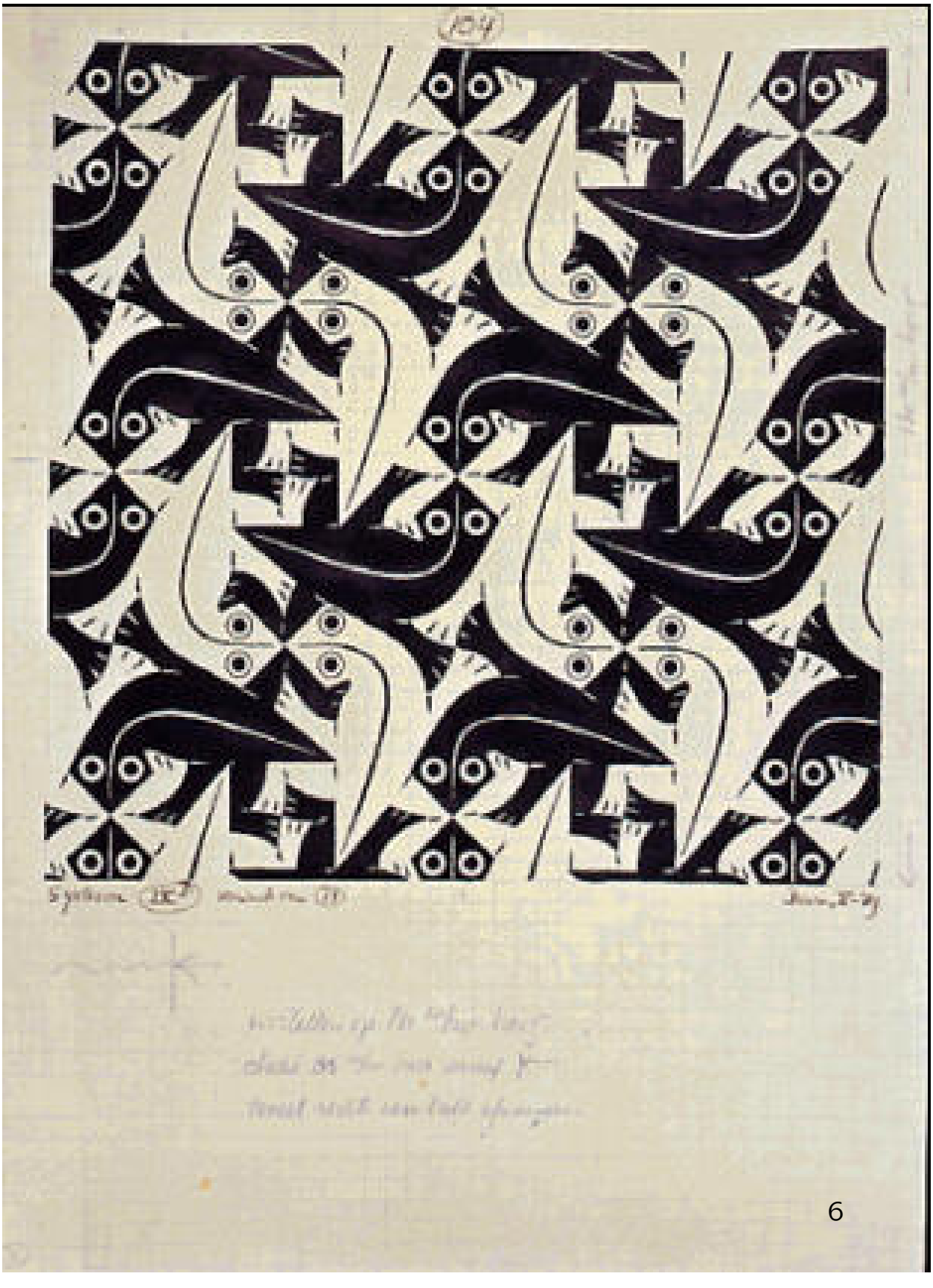
We started with a review of graphs that were symmetric with respect to the x-axis and the origin, and used them to define reflections across a line and rotations around a point. Having just covered polar coordinates, we were able to derive formulas for reflections across lines through the origin and rotations around the origin.

We spent some time composing reflections and rotations in various ways, introducing the notion of a multiplication table, inverses, and order.

Having defined order for symmetries with finite orders, we then talked about translations as examples of symmetries of infinite order.

At this point, we started looking at pictures.





Project:

1. Choose an M. C. Escher drawing from the web site on our class web page. (I am willing to accept other symmetric drawings but will need to see them first to make sure that they are appropriate.) Pretend for the purposes of this project that the drawings extend throughout the plane.

2. Identify which portion of the drawing is being repeated.

3. Identify the symmetries being applied to the given pattern. Your list should be comprehensive in that continual application of the symmetries should yield the entire drawing. The list should be efficient in that you should not include any symmetries that can be gotten by repeated application of the others.

I found that the students in these classes were very enthusiastic about finding all possible symmetries in these Escher drawings. I needed something tangible and engaging to demonstrate our work on reflections, rotations, and translations, and Escher's work definitely served that purpose.

REFERENCES

- Escher, M. C., *The World of M. C. Escher*, New American Library, New York, 1971

- *http://www.mcescher.com*

- Gardner, Martin, *Mathematical Carnival*, Vintage, New York, 1975