Minimal Standards for Writing in Mathematics Courses

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Communication skills are a vital component of mathematics, as few users of mathematics in our society are hermits. Rather than focus on large, term-length writing assignments as a way of developing these skills, I would like to discuss the need to raise the standards of clarity for common, everyday assignments.

A simple example:

Problem: Solve $5x - 2 = 0$.

Solution #1: 5/2

Solution #2: 2/5

Solution #2 is really not much better than Solution #1. It is correct in the sense that if you substituted 2/5 in place of x in the problem as stated, the equation is true. Fortunately there was only one variable, so it was clear how to check the answer. And yet: Why should the reader be the one to check the answer?

The burden of proof should lie with the writer to convince the reader. Many students do not think of themselves as writers when they are in our math classes, and yet almost all of their grades come from written work. That being the case, we need to set standards for writing mathematics.

A more complicated example:

Problem: A spherical balloon is inflated by an air pump at a rate of 100 cubic meters per minute. How big is the radius of the balloon when the radius is increasing at a rate of 2 feet per minute?

Solution #1: 2.0

Who is this answer written for? It does not explain anything. It does not show any steps, nor specify any units. Did the student assume that the answer was 2 feet per minute, as stated in the problem?

As far as I can tell, this answer was written to be compared to a solution key. It was written for someone else to check against a "true" answer that was already written down.

I've spent some time in the private sector, doing financial mathematics. To my knowledge, there never was a solution key for any of the problems that I worked on. The bosses that I gave my work to did not check it for me, nor did they already know the answer. They were paying me to find answers and to convince them that those answers were correct.

Many students stop when they have convinced themselves that their answer is correct. If they are ever going to use their mathematics outside of the classroom, they need to know that that is insufficient. *A problem is solved when you have written an answer that will convince a reasonably intelligent reader.*

Same Problem:

Solution #2: This is a related rates problem where $dV/dt = (dV/dR)(dR/dt)$. We are given that $dV/dt = 100$ cubic meters per minute and that $dR/dt =$ 2 meters per minute. We know from a geometric formula that $V = (4/3)\pi R^3$, so $dV/dR = 4\pi R^2$. Thus $100 = (4\pi R^2)(2) = 8\pi R^2$, $R^2 = 100/8\pi = 4.0$, and $R = 2.0$ meters, where we take the positive square root since R is a distance.

This is a solution that convinces the reader. The only thing that the reader might check further is the arithmetic.

I have given this problem to my calculus class this term. Many of the solutions that came in were like Solution #1. I returned them, and required them to explain their answers in correct English. Not all but many came back looking like Solution #2.

I think that revision is the key. If an answer is not clear enough, the student needs to re-write it. Feedback on clarity can come from the instructor; it can also come from fellow students.

Every couple of weeks in that same calculus class, I have the students work on two separate problems and then switch their papers. Nothing encourages an appreciation for clarity as having to read solutions that lack it.

All of my regular assignments are now revisable, whether the comments come from me or from fellow students. Since it is better to get it right the first time, solutions that need revision are given half credit.

This may sound like extra work, but it really isn't. Solutions that are unclear are returned without any attempt to assign partial credit. Revised work is either easier to read or given a grade of zero.

I am trying to teach my students to perform mathematics in a way that will sustain them outside of my classroom. If I accept less than what I have described above, I believe that I am doing them a disservice.