Computational Group Theory and Symmetry

Jeffrey Clark
Elon University
email: clarkj@elon.edu
web: http://frodo.elon.edu

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Electronic Copy of Talk

This talk can be obtained from my web-site

`http://frodo.elon.edu`

under the link Presentations.

Introduction

This talk will discuss a project that I have used in my introductory abstract algebra class that helps my students make the connection between the symmetry groups of regular polyhedra and their axes of rotation.

Computation in Teaching Mathematics

In courses where the use of modern technology will enhance student learning, departments should adopt methods of teaching mathematical sciences courses that make full use of appropriate current technology.

MAA Guidelines for Programs and Departments
in Undergraduate Mathematical Sciences

Developing Intuition

My students enter my classes with a good deal of intuition about polynomials, trigonometric functions, etc. It is a shock to their system to encounter groups late (typically in their junior year) in their careers and have to develop a sense of how they behave.

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- GAP (Groups, Algorithms, and Programming) is best suited for performing computations of great depth, and is free.
- My class was small—I ran the project off of web page with Perl-based CGI program. In the future I would like to switch to GAP to allow for greater depth of computation in assignments.

Goals of Project

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- Students should understand on a combinatorial level the size of the symmetry groups of the regular polyhedra.
- Students should identify conjugacy classes of symmetries with geometric operations on regular polyhedra.

Project

- Find the conjugacy classes for the symmetry groups of the regular cube, octahedron, dodecahedron, and icosahedron. (The tetrahedron was done in class as an example.)

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- Find the conjugacy classes for the symmetry groups of the regular cube, octahedron, dodecahedron, and icosahedron. (The tetrahedron was done in class as an example.)
- Use the polyhedral dice you've been given to come up with two permutations in cycle notation that you think will generate the symmetry group.

Project (Continued)

- Go to the class web-site, and use the link from the class web page to compute the size of the group generated by your generators. Use the group size that we've computed in class to confirm that the permutations that you use will actually generate the entire group. Then compute the conjugacy classes.

Project (Continued)

- Go to the class web-site, and use the link from the class web page to compute the size of the group generated by your generators. Use the group size that we've computed in class to confirm that the permutations that you use will actually generate the entire group. Then compute the conjugacy classes.
- Once you have identified the conjugacy classes, you are to give a prose description of each class. For example, with the regular tetrahedron, one conjugacy class consisted of setting the tetrahedron on one side and rotating the tetrahedron 120° counterclockwise; another consisted of rotating the tetrahedron 180° around an axis that connects the midpoints of two opposite edges.

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- The students worked on the projects individually, but when they were done I had them present in groups (one for each polyhedron).
- The students were more interested in this project than anything else we did in the semester.

Conclusion

My students have a hard time learning elementary group theory because it appears to be completely abstract to them. To help them, I try to find tangible applications that are accessible to them. The project I described is part of my attempt to make the concepts more intuitive.