Solutions to Exam #4

Math 121-B

Friday, May 5, 2000

1. Horizontal asymptotes correspond to limits at infinity:

$$\begin{aligned} y &= \frac{x+x^2}{3x-4x^2+1} \\ &= \frac{x^2+x}{-4x^2+3x+1} \\ \lim_{x\to\pm\infty} y &= -\frac{1}{4} \\ \text{Horizontal asymptote: } y &= -\frac{1}{4} \end{aligned}$$

2. We split the sum up before simplifying:

$$\begin{aligned} \sum_{i=1}^{n} \frac{3i-2n}{n^2} &= \sum_{i=1}^{n} \frac{3i}{n^2} - \sum_{i=1}^{n} \frac{2n}{n^2} \\ &= \frac{3}{n^2}\sum_{i=1}^{n} i - \frac{2}{n}\sum_{i=1}^{n} 1 \\ &= \frac{3}{n^2}\frac{n(n+1)}{2} - \frac{2}{n}n \\ &= \frac{3(n+1)}{2n} - 2 \\ \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3i-2n}{n^2} &= \frac{3}{2} - 2 \\ &= -\frac{1}{2} \end{aligned}$$

3. We need to express each term as a power of x:

$$\begin{aligned}
\int \left(x^2 + \sqrt{x} - \frac{1}{x^2} - \frac{1}{\sqrt{x}} \right) dx &= \int \left(x^2 + x^{1/2} - x^{-2} - x^{-1/2} \right) dx \\
&= \frac{x^3}{3} + \frac{x^{3/2}}{3/2} - \frac{x^{-1}}{-1} - \frac{x^{1/2}}{1/2} + C \\
&= \frac{x^3}{3} + \frac{2x^{3/2}}{3} + \frac{1}{x} - 2\sqrt{x} + C
\end{aligned}$$

4. We set the area up as a definite integral, and substitute $u = 3x$, $du = 3\,dx$, $dx = du/3$:

$$\begin{aligned}
\text{Area} &= \int_{\pi/6}^{\pi/3} \sin(3x)\,dx \\
&= \int_{\pi/2}^{\pi} \sin(u) \frac{du}{3} \\
&= \frac{1}{3}(-\cos(u))\Big|_{\pi/2}^{\pi} \\
&= \frac{1}{3}(-\cos(\pi) - (-\cos\left(\frac{\pi}{2}\right))) \\
&= \frac{1}{3}(-(-1) - 0) \\
&= \frac{1}{3}
\end{aligned}$$

5. The Fundamental Theorem of Calculus is used to evaluate definite integrals: if you can find an antiderivative for the integrand, then plug the limits into the antiderivative and take the difference; the result is the value of the definite integral.

6. Let x be the price of the car. We use the point-slope form of the equation of a line to find the sales formula:

$$\begin{aligned}
\text{sales} - 40 &= \left(-\frac{2}{1000}\right)(x - 18{,}000) \\
\text{sales} &= 40 - 0.002x + 36 \\
&= -0.002x + 76 \\
\text{revenue} &= x(\text{sales}) \\
&= -0.002x^2 + 76x \\
(\text{revenue})' &= -0.004x + 76 \\
\text{Critical value: } 0 &= -0.004x + 76 \\
0.004x &= 76
\end{aligned}$$

$$
\begin{aligned}
x &= \frac{76}{0.004} \\
&= 19{,}000 \\
(\text{revenue})'' &= -.008 \text{ indicates a maximum}
\end{aligned}
$$

and the dealership would make more money by selling the car at a price of \$19,000; they would sell 38 cars for a revenue of \$722,000, as opposed to their current revenue of 40(\$18,000) = \$720,000.

7. We take the indefinite integral, then use our initial condition to solve for C:

$$
\begin{aligned}
f(x) &= \int f'(x)\,dx \\
&= \int \sqrt{x}\,dx \\
&= \int x^{1/2}\,dx \\
&= \frac{x^{3/2}}{3/2} + C \\
&= \frac{2x^{3/2}}{3} + C \\
f(9) &= 5 \\
\frac{2 \cdot 9^{3/2}}{3} + C &= 5 \\
\frac{2 \cdot 27}{3} + C &= 5 \\
18 + C &= 5 \\
C &= -13 \\
f(x) &= \frac{2x^{3/2}}{3} - 13
\end{aligned}
$$

8. We will use the substitution $u = x^2 + x - 2$, $du = (2x+1)\,dx$:

$$
\begin{aligned}
\int \frac{2x+1}{(x^2+x-2)^5}\,dx &= \int u^{-5}\,du \\
&= \frac{u^{-4}}{-4} + C \\
&= -\frac{1}{4(x^2+x-2)^4} + C
\end{aligned}
$$

9. You need to identify a piece of the integrand to replace by a new variable such as u. If the integrand involves a trigonometric function of another

function, then letting u be the angle often works. Another good strategy is if you have a function raised to a power (including roots) in your integrand, let u be that function.

You also need to calculate du and make sure that it appears in your new integral. If all goes well, you should end up with a simpler integral in terms of u. Once you have solved it, replace u by what it stands for, so that your answer is in the same variable that you started with.

10. The sum is over three terms ($i = 3, 4, 5$):

$$\begin{aligned}
\sum_{i=3}^{5} \frac{1}{i^2} &= \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} \\
&= \frac{1}{9} + \frac{1}{16} + \frac{1}{25} \\
&= \frac{400}{3600} + \frac{225}{3600} + \frac{144}{3600} \\
&= \frac{769}{3600} \\
&= 0.214
\end{aligned}$$