

Exam #2

Math 312-A

Thursday, April 6, 2000

For full credit, explain all of your steps. You may answer the questions in any order as long as you label your work clearly. Put your answers on the blank paper, and staple the questions on top when you are done.

1. Find all of the cosets of H in G , where

$$H = \{(1), (1\ 3)(2\ 4)\}$$

is a subgroup of

$$G = \{(1), (1\ 2\ 3\ 4), (1\ 3)(2\ 4), (1\ 4\ 3\ 2), (1\ 2)(3\ 4), (1\ 4)(2\ 3), (2\ 4), (1\ 3)\}$$

2. Show that if y is the conjugate of x by g , then y has the same order as x .
3. Let

$$H = \left\{ \left(\begin{array}{ccc} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{array} \right) \mid x, y, z \in \mathbb{R} \right\}$$

Show that H is a subgroup of $GL_3(\mathbb{R})$ (the set of all invertible 3×3 matrices with real elements).

You may use the fact that

$$\left(\begin{array}{ccc} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{array} \right)^{-1} = \left(\begin{array}{ccc} 1 & -x & xz - y \\ 0 & 1 & -z \\ 0 & 0 & 1 \end{array} \right)$$

4. Define $f : S_3 \rightarrow A_3$ by $f((1\ 2\ 3)) = (1\ 2\ 3)$, $f((1\ 3\ 2)) = (1\ 3\ 2)$, and $f(x) = (1)$ for all other $x \in S_3$. Prove or disprove that f is a homomorphism.

5. Given that $H = \{\pm I\}$ is normal in Q_8 , and the following multiplication table for Q_8 , write out a multiplication table for the quotient group Q_8/H .

	I	$-I$	J	$-J$	K	$-K$	L	$-L$
I	I	$-I$	J	$-J$	K	$-K$	L	$-L$
$-I$	$-I$	I	$-J$	J	$-K$	K	$-L$	L
J	J	$-J$	$-I$	I	L	$-L$	$-K$	K
$-J$	$-J$	J	I	$-I$	$-L$	L	K	$-K$
K	K	$-K$	$-L$	L	$-I$	I	J	$-J$
$-K$	$-K$	K	L	$-L$	I	$-I$	$-J$	J
L	L	$-L$	K	$-K$	$-J$	J	$-I$	I
$-L$	$-L$	L	$-K$	K	J	$-J$	I	$-I$