Exam #2 Solutions

Math 114-F

Friday, March 15, 2002

1. We will use the Multiplication Rule.

   There are four aces out of fifty-two cards, so the probability that the first card is an ace is 4/52.

   Given that the first card is an ace, there are four two's out of fifty-one cards, so the probability that the second card is a two (given that the first was an ace) is 4/51.

   Given that the first two cards were an ace and a two, there are four three's out of fifty cards, so the probability that the third card is a three (given that the first two cards were an ace and a two) is 4/50.

   The probability of dealing out an ace, then a two, and then a three is:

   $$\frac{4}{52}\frac{4}{51}\frac{4}{50} = 0.00048 = 0.048\%$$

2. A discrete random variable is one that can only take whole number values, such as the number of states that a person has visited. A continuous random variable is one that can take a range of values (including fractions), such as a person's height.

3. The probability of getting at least one head can be found from the probability of getting no heads. Using the multiplication rule, the probability of getting tails on each of ten tosses is $(1/2)^{10} = 0.001$. The probability of getting at least one head is $1 - 0.001 = 0.999 = 99.9\%$.

4. X is a binomial random variable with $n = 15$ and $p = 0.6$. The mean of X is $\mu = np = (15)(0.6) = 9$ and the standard deviation of X is $\sigma = \sqrt{npq} = \sqrt{(15)(0.6)(0.4)} = 1.90$.

5. The slips are chosen without replacement and the order does matter, so we will use permutations to count how many ways the prizes could be awarded.

   There are ${}_{30}P_3 = 24{,}360$ ways that the prizes could be awarded.

6. There are $500 + 3500 = 4000$ people with college degrees in the table; of them 500 have a graduate degree. The probability of someone with a college degree having a graduate degree is $500/4000 = 12.5\%$.

7. We extend the table to find the mean and variance:

x	$P(x)$	$xP(x)$	$(x-\mu)^2P(x)$
2	0.2	0.4	0.8
3	0.5	1.5	0.5
7	0.3	2.1	2.7
		$\mu = 4.0$	$\sigma^2 = 4.0$

and $\sigma = \sqrt{4.0} = 2.0$.

8. The number of people in the survey that are left-handed X is a binomial random variable: the sample is a random one and the probability of any one person being left-handed is fixed at $p = 10\% = 0.1$. The sample size is 25, and we want to know the probability that $X = 3$, which is `binompdf(25, 0.1, 3)` $= 0.226 = 22.6\%$.

9. Over repeated measurements, X takes the value 1 99% of the time, and takes the value 3 only 1% of the time. Consequently the average should be much closer to 1 than to 3, and in fact $\mu = 1(0.99) + 3(0.01) = 1.02$.

10. We need to check whether or not $P(A \text{ and } B) = P(A)P(B)$. Since there is only one way for A and B to be true (the roll must be $2-2$), $P(A \text{ and } B) = 1/36$. Since there are six ways to roll doubles, $P(A) = 6/36$. Since there are three ways ($1-3$, $2-2$, $3-1$) to roll a four, $P(B) = 3/36$. Since $1/36 \neq (6/36)(3/36)$, the two events are dependent.

    Why should this be so? You can't roll doubles and have the sum be any of the odd numbers 3, 5, 7, 9, or 11. If you know that you have rolled doubles, it increases the chances of rolling any of the even sums, and thus the chance of rolling a four.