On my honor, I have abided
by the Elon University Honor Code

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Name Signature

Final Exam

Math 321-A

Thursday, May 15, 2008

For full credit show all work. When in doubt, explain your reasoning.

Find all integrals numerically; you need not find an exact solution to them. Round answers to the third decimal place. Find all angles in radians.

1. Find and sort any critical point(s) of the function $f(x, y) = 2x^2 + 10xy + 13y^2 + 22x + 58y$.
2. Find dy/dx using partial derivatives for the following curve:
$$x^3 + x^2y + xy^2 + y^3 = 4$$
3. Write the previous equation in polar coordinates and simplify as far as possible.
4. Summarize how we compute the area for a surface given parametrically and why it works.
5. What makes a point on a surface $z = f(x, y)$ a saddle point, i.e., what is the definition in your own words of a saddle point?
6. Let $\vec{r}(t)$ be a differentiable curve whose speed is always positive.
   (a) What does it mean for $\vec{T}$, $\vec{N}$, and $\vec{B}$ when the curve is a straight line?
   (b) What does it mean for $\vec{T}$, $\vec{N}$, and $\vec{B}$ when the curve lies in a fixed plane?
7. Find the mass of the hemisphere $R = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 4, z \geq 0\}$ given the density function $f(x, y, z) = z^2$ using spherical coordinates.
8. Let $\vec{r}(t) = \langle a\cos(t), a\sin(t)\rangle$ be a circle of radius $a > 0$, centered at the origin. Show that the curvature of the circle is a constant, and compute it.
9. Find the rate of change for $f(x, y, z) = xy/z$ at the point $(-2, 3, 7)$ in the direction $\langle 1, 2, -4\rangle$.
10. Describe the information about a vector field provided by its curl.
11. Find the coordinates of the foci for the following ellipse:
$$\frac{(x+3)^2}{9} + \frac{(y-1)^2}{49} = 1$$
12. Graph the following curve and find the area that it bounds.
$$r = 2 + \cos(3\theta) \qquad 0 \leq \theta \leq 2\pi$$
13. Find the length of the curve in the previous question. (Numerically integrate—you won't be able to do this one by hand.)

14. At a point on a curve, $\overrightarrow{v} = \langle 9, 7, 0\rangle$, $\overrightarrow{a} = \langle -6, 4, -7\rangle$, and $\overrightarrow{jerk} = \langle 7, 3, -1\rangle$. Find the torsion of the curve at that point. Can you tell if the curve is planar from your answer?

15. Find the tangential and normal components of acceleration for $\overrightarrow{r}(t) = \langle t^2 - 1, t^3 - t\rangle$. (Note: the cross product of $\langle a, b, 0\rangle$ with $\langle c, d, 0\rangle$ is $\langle 0, 0, ad - bc\rangle$.

16. Construct an equation for the tangent plane to the ellipsoid $x^2 + 2y^2 + 3z^2 = 300$ at the point $P = (-6, -6, -8)$.

17. Find the perpendicular distance between the following two lines.

$$\text{Line \#1: } (x, y, z) = (3 - 2s, 4 + 5s, 6 + s)$$
$$\text{Line \#2: } (x, y, z) = (5 - t, 7 - t, 1 + t)$$

18. Find an equation for the plane through the following three points;

$$P = (1, 7, -8)$$
$$Q = (-4, 4, 1)$$
$$R = (3, -9, -8)$$

19. Given that $\overrightarrow{A}$ and $\overrightarrow{B}$ are non-zero vectors that are not parallel, what can you say about the direction of $\overrightarrow{A} \times \left(\overrightarrow{A} \times \overrightarrow{B}\right)$?

20. We know from our work with Kepler's Laws that planets and returning comets travel in ellipses around our sun, while non-returning comets travel in hyperbolic orbits. Why are there no parabolic orbits?