Project #3

Math 321-A

Due Friday, May 9, 2008

Pappus Theorem states that for a surface of revolution, the surface area is the product of the length of the curve with the distance traveled by the centroid, i.e., the length of the curve with 2π times the distance from the centroid to the axis of revolution.

In this project you will verify Pappus Theorem for your curve as it is rotated around the x-axis.

Some of your integrations will generate a warning about loss of precision; don't worry about it.

Your project should be submitted as a `Mathematica` notebook electronically. It should include the following, all appropriately formatted:

1. Title and name at the top.
2. Section titles for each of the following:
   (a) Description of the problem.
   (b) Defining and plotting the curve `r[t]`. Let `f[t]` and `g[t]` be the x and y coordinates of `r[t]` respectively.
   (c) Find the arc-length L numerically.
   (d) Find $M_y = \int x\, ds = \int_a^b f(t)\frac{ds}{dt}\, dt$ numerically.
   (e) Find $M_x = \int y\, ds = \int_a^b g(t)\frac{ds}{dt}\, dt$ numerically.
   (f) Find the centroid $(\bar{x}, \bar{y}) = (M_y/L, M_x/L)$ for your curve.
   (g) Define a surface of revolution around the x-axis:
   `surf[t_,u_] = {f[t], g[t]*Cos[u], g[t]*Sin[u]}`
   where $0 \le u \le 2\pi$.
   (h) Plot the surface.
   (i) Compute the surface area for your surface of revolution numerically.
   (j) Compute the product of the length of your curve with the distance traveled by the centroid, i.e., $L(2\pi\bar{y})$. (This should turn out to be the same number as the surface area.)